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A percolation model of innovation in complex technology spaces

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Abstract

Innovations are known to arrive more highly clustered than if they were purely random. Their distribution of importance is highly skewed and appears to obey a power law or lognormal distribution. Technological change has been seen by many scholars as following technological trajectories and being subject to ‘paradigm’ shifts from time to time.

To address these empirical observations, we introduce a complex technology space based on percolation theory. This space is searched randomly in local neighborhoods of the current best-practice frontier. Numerical simulations demonstrate that with increasing radius of search, the probability of becoming deadlocked declines and the mean rate of innovation increases until a plateau is reached. However, for ‘richer’ technological environments, a ‘trough’ separates myopic from long-range search due to the effect of R&D duplication. The distribution of innovation sizes is highly skewed and may resemble a Pareto distribution near the critical percolation probability.

JEL classification: C15; C63; D83; O31

Keywords: Innovation; Percolation; Search; Technological change; R&D

1. Introduction

While we like to think of innovations as distinct, easily identifiable entities, closer inspection reveals that they are anything but: they can be resolved into smaller sub-steps, making the definition somewhat arbitrary. Nevertheless, when the minimum number of essential subunits comes together, one does have the feeling that the innovation ‘pops

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out' and becomes a recognizable Gestalt. Thus a seemingly simple innovation such as the bicycle is a concatenation of many sub-innovations spread out over time:

> In 1818, K.V. Drais de Sauerborn presented his Draisine, a kind of walk-drive bicycle (Laufrad). In 1839, Mannilau demonstrated how wheels can be driven by pedals, and in 1861 at the latest pedals were built into the Draisine. In 1867, they were used on the front wheel by Michaux, and during the next few years the bicycle industry in France grew rapidly. A model of the bicycle approaching the one we are accustomed to today was constructed by Lawson in 1879, but a commercially successful 'safety bike' was not introduced by Starley until 1885. If we take 1818, 1839 or 1861 alternatively as years of invention, and 1867, 1879 or 1885 alternatively as years of basic innovation, we can obtain nine different results for the time-span between invention and innovation. [Brockhoff (1972, p. 283), cited by Kleinknecht (1987, p. 61)]

Undoubtedly, numerous other examples could be found in the history of technology to reinforce this point. What we normally perceive as a unitary entity, a radical innovation, in reality is usually composed of a number of smaller steps dispersed in time, often involving borrowing from other fields or dependent on specific unrelated advances in order to make the final step possible. In the bicycle case we could add the availability of pneumatic tires and ball bearings (and thus precision machining, the precision grinding machine, etc.) as essential complementary innovations without which the bicycle boom of the 1890s would have been unthinkable. The bicycle is not one innovation but a succession of several smaller ones. In fact, our problem is not reducible à la Schumpeter to just radical vs. incremental innovations; rather innovations come in all sizes, suggesting a fractal structure to the process of innovation.

This ambiguity regarding the timing and definition of innovations is not merely a matter of historical curiosity. It can also be profitably exploited in a representation of technology as consisting of a multitude of elemental small inventive steps that must come together, much like the pieces of a mosaic, to form a coherent whole and constitute an innovation. The purpose of this paper is to present a model of the dynamics of this process making as few assumption about the nature of technology as possible except that it is in some sense complex and shrouded in uncertainty.

The paper is organized as follows. In Section 2 we briefly present some stylized facts about technical change and innovation and some empirical data highlighting a number of distinctive statistical patterns associated with the innovative process. Section 3 outlines the framework of the model, which is derived from Silverberg (2002). Section 4 presents the results of extensive numerical simulations. We propose more sophisticated search strategies in Section 5 and draw some conclusions.

2. Stylized facts about innovation and technological change

The innovation process, based as it is on the discovery of the genuinely novel, is fraught with true Knightian uncertainty. If we could predict an innovation in detail in advance, it would not be novel. Nevertheless, innovations do not appear to be completely random and unrelated. This has lead to the identification of a number of stylized facts about the innovation process (see e.g. Dosi, 1988), some of which have been substantiated quantitatively while others are still only impressionistic hypotheses. Our model is inspired in its basic assumptions by some of these facts and in its implications is intended to address a number of other facts (as well as providing a more concrete and quantitative framework for discussing and elucidating the debate surrounding the 'inspiring' facts). As inputs we take the following stylized facts:

- Technical change is cumulative: new technologies build on previous discoveries and often draw on advances in seemingly unrelated fields. For example, Edison's electric light presupposed both advances in the generation of electricity, the manufacture of conducting filaments, and improvements in vacuum pump technology.
- The arrival of innovations appears to be a stochastic process. Schumpeter (1939) initiated a debate about whether the arrival of 'major innovations' (in the sense of their economic impact) is clustered in time. In different papers (Silverberg and Lehnert, 1993; Silverberg and Verspagen, 2003a) we used a time-homogenous Poisson process as the benchmark against which to evaluate the data-generating properties of these major innovation time series. The evidence suggests both long-run trends in the arrival rate, and a distribution with significant overdispersion. The latter implies that there are randomly distributed periods of relatively (compared to a Poisson) high and low activity in the time series for major innovations, or, in other words, temporal clustering.
- Agents tend to search locally for new technologies, i.e., they try combinations and extensions of existing knowledge close in some space of technological characteristics to what they already know and use (cf. Atkinson and Stiglitz, 1969; Nelson and Winter, 1977, 1982).
- Technical change follows relatively ordered pathways, as can be measured ex post in a space of technological characteristics. Examples of propositions in this direction are Nelson and Winter's (1977) *natural trajectories*, Sahal's (1981) *technological guideposts*, and Dosi's (1982) *technological paradigms*. These concepts are often used to explain the specific direction in which a technology develops after an initial radical breakthrough takes place. The factors that may influence such a trajectory are incremental improvements that take place during the diffusion process of the basic design, and external circumstances such as characteristics of demand, factor prices, patterns of industrial conflict, etc., but there is also an underlying presupposition that trajectories only move within a relatively small range of 'naturally' preordained channels. Dosi (1982) describes the result of these considerations as a "model and pattern of solution of selected technological problems, based on selected principles from the natural sciences and on selected material technologies". From all the possible directions technological development may take, only a small portion are realized.

Radical improvements in the performance of a technology are often related to a change of trajectory. An example of this is provided by the long-term history of

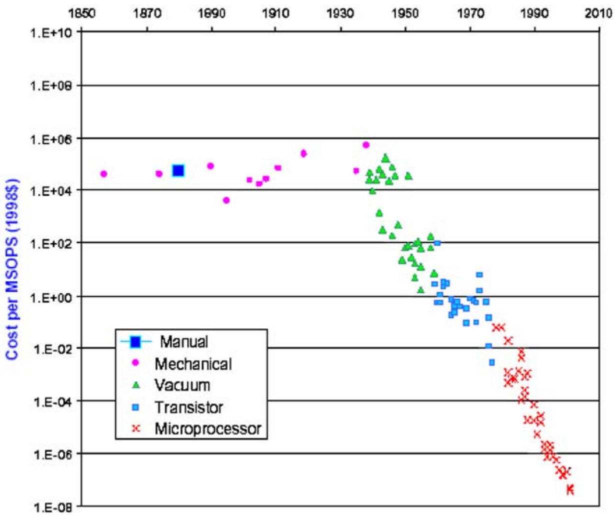

Fig. 1. The real cost of computation per million standardized operations for different technologies. The paradigm shift in the 1940s is apparent. Source: Nordhaus (2001).

computing technologies, where Nordhaus (2001) has compiled an indicator of technological performance. The essentially static trajectory of manual and mechanical computation was replaced by an exponentially changing electronic trajectory (which in turn went through several generations of underlying component technologies) after the later 1930s (see Fig. 1).

Changes in trajectories are often the result of bifurcation or merging. For example, Foray and Grübler (1990) show how in the field of moldings for ferrous castings, the so-called gasifiable pattern processes broke away as a process simplification from the established sand molding trajectory. This was associated, however, with an increase of technical complexity, and led to markedly different diffusion rates in France and Germany. Along these lines, many authors have suggested that a certain arbitrariness exists in the path actually chosen, which could be the result of small random events (as in path dependence, see Arthur, 1994, or genetic drift, see Kimura, 1983 on molecular evolution) and cultural and institutional biases (e.g., in the theory of social construction of technology discussed in Bijker et al., 1987).

While the above stylized facts are taken as an input to our modeling strategy, the simulation results documented below point to a further issue analyzed in the recent literature, i.e., the size distribution of innovations. This distribution has been shown to be highly skewed (with a preponderance of small innovations), and possibly heavy-tailed with a power-law character (linear on a log–log plot). The measures that have been employed to quantify these characteristics are citation and co-citation frequencies (e.g., Trajtenberg, 1990; van Raan, 1990), and innovation returns data (e.g., Scherer 1998; Harhoff et al., 1999; Scherer et al., 2000). Fig. 2 presents the raw Trajtenberg CT-scanner patent citation data, while Fig. 3 transforms them into a so-called

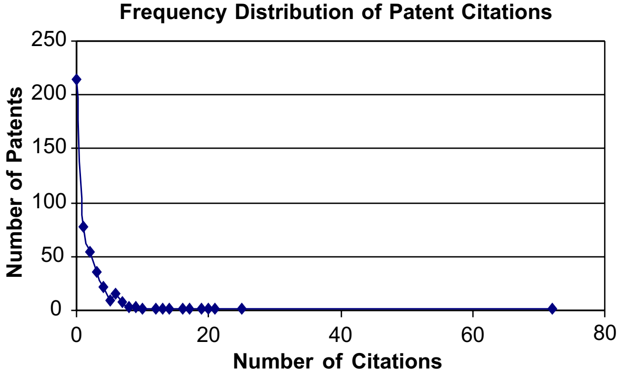

Fig. 2. Innovation 'size' distributions based on CT scanner patent citations, by number of citations.

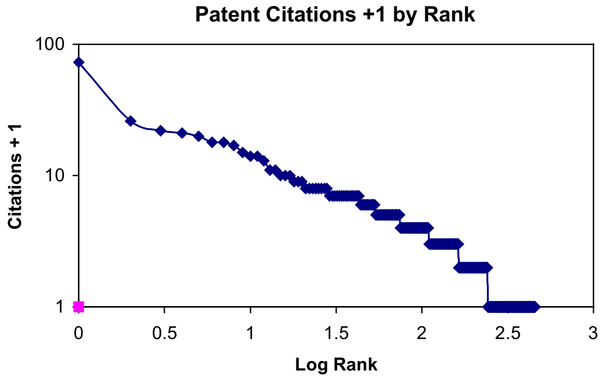

Fig. 3. Innovation 'size' distributions based on CT scanner patent citations, rank-order distribution counting self-citation, double-log scale.

Zipf plot or rank-order distribution. A linear curve in the double-log plot corresponds to a power-law of the rank-order distribution, which seems to be approximately the case for these data. The innovation-returns distributions, on the other hand, seem to be situated somewhere between a lognormal and a true Pareto power law.

Until now, no convincing explanation has been proposed to explain the extreme skewness of these distributions. Since their high variance and skewness (in the case of a Pareto, infinite variance and even infinite expectation for some parameter values) has important implications for the risk management of R&D investment policy, an understanding of this phenomenon and its underlying causes seems highly desirable (see, e.g., Scherer and Harhoff, 2000). It also has striking implications for the variability of economic growth rates (Nordhaus, 1989; Sornette and Zajdenweber, 1999).

3. Technology space as a percolated lattice and R&D as stochastic interface growth

Consider a lattice, unbounded in the vertical dimension, anchored on a baseline (or space), with periodic boundary conditions, as in Fig. 4. The horizontal space represents the universe of technological niches, with neighboring sites being closely related. While the technology space is represented here and in the following as one-dimensional (with periodic boundary conditions, i.e., a circle), it can easily be generalized to higher dimensions or different topologies. The vertical axis measures an indicator of performance intrinsic to that technology and could also be conceived as multidimensional. For simplicity, we will restrict ourselves to a two-dimensional lattice in the following.

A lattice site a_{ij} can be in one of four states: 0 or technologically excluded by nature, 1 or possible but not yet discovered, 2 discovered but not yet viable, and 3, discovered and viable. A site moves from state 2 to 3, from discovered to viable, when there exists a contiguous path of discovered or viable sites connecting it to the baseline (see Fig. 4). The neighborhood relation we shall use is the von Neumann one of the four sites top, bottom, right and left $\{a_{i\pm1,j}, a_{i,j\pm1}\}$, with periodic boundary conditions horizontally. The intuition here is that a discovered technology only becomes viable or operational when it can draw on an unbroken chain of supporting technologies already in use. Until such a chain is completed, the technology is still considered to be under development—it is still an invention, not an innovation. Impossible states 0 remain so forever. State 1 can progress to state 2 if it is uncovered by the R&D search process, and state 2 can possibly but not necessarily progress to state 3 if a connecting chain exists and all its links are discovered.

The lattice dynamics result from the interplay of natural law with the history of human-driven technological search. Two extreme views stake out the range of approaches now current in technology studies, while a third represents a kind of

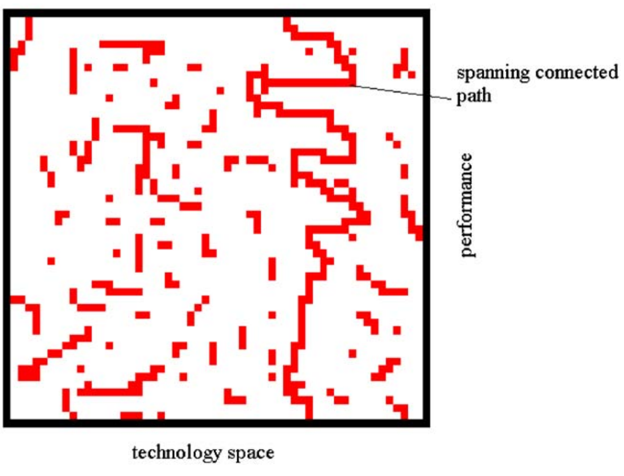

Fig. 4. Technology–performance lattice. Discovered sites are marked in gray, viable sites lie on the path connected to the baseline.

philosophical compromise between the two:

(1) The *social construction of technology* (*SCOT*) perspective says that any site we try is valid technological knowledge that can potentially be incorporated into a viable technology. Thus in this case, a tried site will immediately become occupied and placed in state 2. The paths that result from innovative search will be pure accidents of history.
(2) The alternative *technological determinism* (*TD*) perspective says that a tested site only represents true technological knowledge if it accords with the a priori underlying laws of nature. Thus when we 'invent' a site, we must first test whether it is technologically possible (in state 1). If it is, we raise it to state 2, if not, we leave it in state 0. This is a bit like playing the game minesweeper. The paths that result will be a selection from the technologically possible ones.
(3) A compromise view, which we shall call the *nothing is impossible at a price* (*NIP*) perspective, holds that any site can become viable if we are willing to invest sufficiently to develop it. The development costs can be a random variable between 0 and ∞. The best-practice frontier (BPF) (defined below) will advance at the points of least resistance and often be delayed until sufficient resources can be brought to bear against obstacles. The dynamics may resemble the self-organized criticality observed in the Sneppen (1992) model of 'pinning' interface growth.

If we are willing to allow for natural law, we must first initialize the lattice at time 0 by assigning each site the state 0 or 1. To reflect our a priori ignorance of the laws of nature we regard this as a random process creating a percolation on the lattice with some probability q.[1] The essential property of percolation is the behavior of connected sets as a function of the (uniform and independent) probability of occupation of sites. On an infinite lattice (including the half-plane) there exists a threshold probability p_c below which there is no infinite connected set and above which with probability one there is one (and only one) infinite connected set. The probability that any site will belong to the infinite connected set is obviously zero below p_c and increases continuously and monotonically above p_c (Fig. 5).[2] For bounded lattices such as in Fig. 4, the interesting question is the probability of finding a connected path spanning the lattice from the bottom edge to the top one. This will increase rapidly and non-linearly in the neighborhood of p_c. A metaphor that may help to sharpen intuition is to regard rain falling on a yard as a percolation problem. After only a bit of rain the yard consists of islands of wetness surrounded by dry pavement. After more rain has fallen

[1] In this case we speak of site percolation, as opposed to working with the lines connecting nodes, known as *bond* percolation (see Grimmett, 1989; Stauffer and Aharony, 1994). For the purposes of this paper there is no obvious preference for one or the other (and bond percolation can always be reformulated as a site model). An early application of percolation theory to technological change can be found in Cohendet and Zuscovitch (1982). David and Foray (1994) applied a hybrid site and bond percolation model to the standardization and diffusion problem in electronic data interchange networks. Some recent applications of percolation theory to social science problems include Solomon et al. (1999), Goldenberg et al. (2000), Gupta and Stauffer (2000) and Huang (2000).

[2] For bond percolation on the unbounded plane it can be proven that p_c is exactly $\frac{1}{2}$. For site percolation it has been numerically established to be around 0.59. See Grimmett (1989) and Stauffer and Aharony (1994).

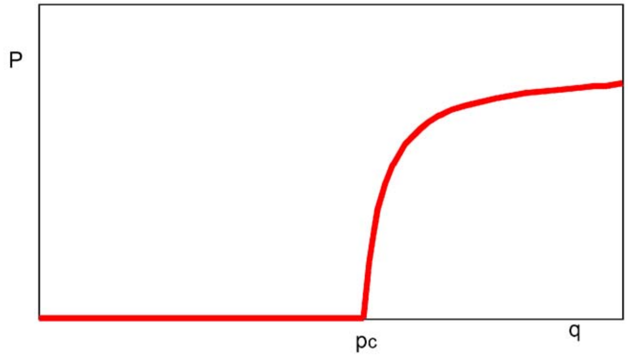

Fig. 5. The probability that any site will be on the infinite cluster p as a function of the percolation probability q.

the yard suddenly flips to being islands of dryness surrounded by wetness. Regarding technology space as a percolation is of course only one way to generate a 'complex' problem setting. Other possibilities are the use of NK-landscapes (see e.g. Frenken, 2001; Kauffman et al., 2000) or directed networks (Vega-Redondo, 1994, although networks can also be used as the substrate for percolated structures). In a future paper we hope to relate our approach to landscape models in more detail, possibly by replacing the regular lattice of our present technological baseline with the graph induced by a genetical representation of technologies as strings of binary digits.

If $q < p_c$, then there will only be finite connected sets (clusters) and technological change will eventually come to an end. If, however, nature is so bountiful that $q > p_c$, then there will be a unique infinite cluster and thus potentially unbounded paths of innovation. And the larger q, the denser the network of potentially viable technologies will be. The social construction of technology case results from technological determinism in the limit $q \to 1$.

We now come to the R&D search half of the dynamics. At any point in time t a BPF can be defined consisting of the highest sites in state 3 for each baseline column (of which there are N_c):

$$BPF(t) = \{(i, j(i)), i = 1, N_c\} \quad \text{where } j(i) = (\max j | a_{i,j} = 3).$$

(If there is no viable site in column i^* we set $j(i^*) = -1$.) The $BPF(t)$ is needed as the anchor for the R&D search process, which is characterized by a search radius m. Around each point $(i, j) \in BPF(t)$ with $j > -1$, i.e., around each occupied point on the frontier, we draw a (diamond-shaped) neighborhood of radius m containing all points at a distance of m or less (according to the 'Manhattan' metric induced by the neighborhood relation). We suppose R&D search to proceed within these local neighborhoods anchored around current best practice, and thus includes technology sites not only directly above the current best-practice sites, but sites laterally related to it and even sites lying behind it. Search itself is viewed as uncertain and characterized by a uniform probability p_s of testing any one of the $2m(m + 1)$ neighboring points

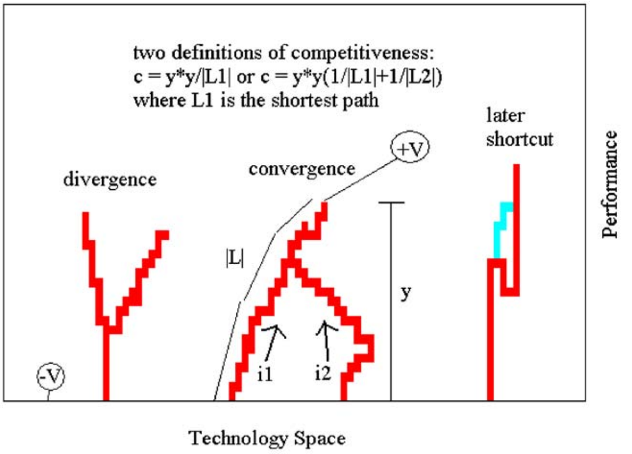

Fig. 6. Diverging technological trajectories on the left, converging ones in the middle. 'Pruning' of paths leads to incremental innovation (right). The economic competitiveness is the square of technological performance divided by path length.

(not counting the anchor point). If the total R&D 'effort' at the disposal of any point on the BPF is E, then

$$p_s = E/2m(m+1).$$

If a site is tested and in state 0, i.e., it is intrinsically impossible, then it remains in this state. If it is in state 1 it is marked as 'discovered' and advanced to state 2. Sites already in state 2 or 3 remain unchanged. A site may be tested several times in a period if it is in the m-neighborhood of several sites on the BPF.

Fig. 6 shows how connected paths may represent some relevant technological phenomena. First, any connected path beginning on the bottom line can be thought of as a natural trajectory. On the left, we see two trajectories diverging from a common origin. In the middle we see technological convergence (e.g., the convergence of mechanical and electronic technologies to mechatronics, or optical and mechanical technologies to optronics). While the purely technological performance characteristics of an operational site are measured by its height above the baseline, its economically relevant *technological competitiveness* can be measured in different ways. The point of introducing a separate technological competitiveness is to reflect the ease of realization (related to cost) of a given level of technological performance and allow subsequent incremental innovations to operate. Additionally, we may want the extent of parallelism in the realization of a technology to be counted as an advantage. Thus we propose two separate measures of competitiveness, both based on path length (if L is a path then let $|L|$ be its length). The first measure is

$$c_1 = y^2/|L_s|,$$

where L_s is the shortest path connecting the site to the baseline and y is the height of the site. If this path is simply a straight vertical line, then $c_1 = y$. The more indirect

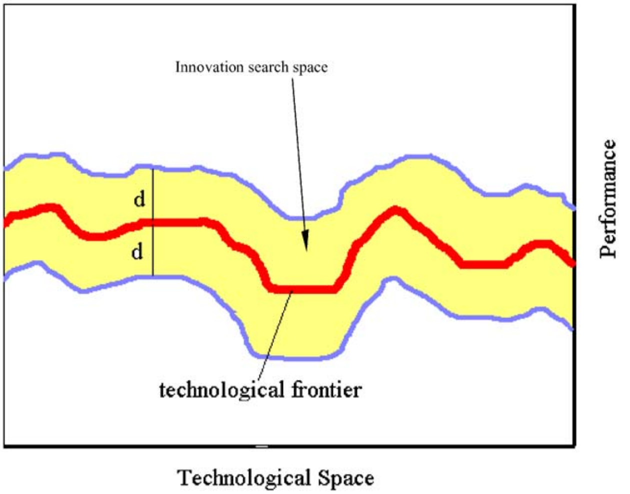

Fig. 7. Search neighborhoods of radius d define an innovation search space enveloping the BPF.

the path, the more the competitiveness is diminished. The second measure corresponds to the current that would be extracted at the site if we apply a one-volt potential difference between the site and the baseline and set the resistance of a single lattice nearest-neighbor link to one. If two paths L_1 and L_2 converge at a site, then

$$c_2 = y^2(1/|L_1| + 1/|L_2|).$$

For more complicated connections Kirchhoff's laws have to be applied.

A relevant technological analogy would be the different generations of microprocessors. While each generation represents a certain gain in performance, it usually comes at a certain price. However, over time that price declines as learning takes place in the production and design of the product. This can be captured in a natural way in our framework by allowing subsequent shortcuts (which we identify with *incremental* innovation) to reduce the length of the connecting base of a site (rightmost in Fig. 6). Thus we will allow innovation to take place both ahead and behind of the current BPF, so that radical and incremental innovation take place simultaneously.

Consistent with our 'blunderbuss' vision of the search process, we allow innovation to take place in a neighborhood of radius m centered around each point on the frontier. The union of these regions creates a band of innovative percolation extending ahead and behind the frontier (Fig. 7). Within this region new sites will be tested at random with some probability p. A discovered site of course need not connect immediately with the operational network. It is this fact that permits innovations of variable length (as measured by the jump in y they entail) to occur spontaneously. Thus we obtain a natural explanation of innovation clustering (but of the random kind), as shown in Fig. 8. This happens when a disjoint extended network of discovered but not yet operational sites is finally connected to the technological frontier, and/or when an 'overhanging

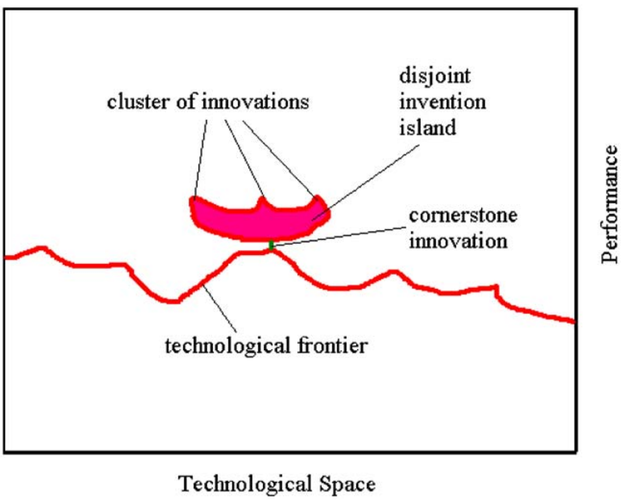

Fig. 8. Clusters of innovations occur when disconnected islands of inventions are joined to the BPF by cornerstone innovations.

cliff' advances laterally, pulling up the BPF at neighboring sites by increments that can be much larger than m, the search radius.

4. Numerical simulations

In the following, we investigate the behavior of the system just described as a function of certain key exogenous parameters, in particular the search radius m and the lattice percolation probability q. We will focus solely on changes in pure technological performance, leaving economic performance, incremental innovation and learning curves for a later paper. We hold the number of columns N_c fixed at 100 and set the total search effort E_0 to 0.05. The vertical dimension can be allowed to grow over time without limit without exceeding the memory capacity of the computer by simply following a band on the lattice around the BPF whose height is greater than $\max_j BPF(t) - \min_j BPF(t) + 2m$. The system is set in motion after percolation by randomly seeding the baseline with 'discovered' sites, that is, turning baseline sites in state 1 into 2's with probability 0.5.

Fig. 9 presents a screen shot of the computer program. Black squares represent the excluded or undiscoverable sites, light disks the already discovered sites and white ones the viable sites. The dark line is the BPF. The graph of the viable sites displays the generic mushroom cloud shape that constantly reappears in the evolution of the system: a narrow stem supporting overhanging cliffs. At the edge of the cliffs the BPF makes large (and in fact potentially unboundedly so) jumps in height, representing technological breakthroughs that have not been achieved by a direct approach but rather by taking a detour through distant technological regions.

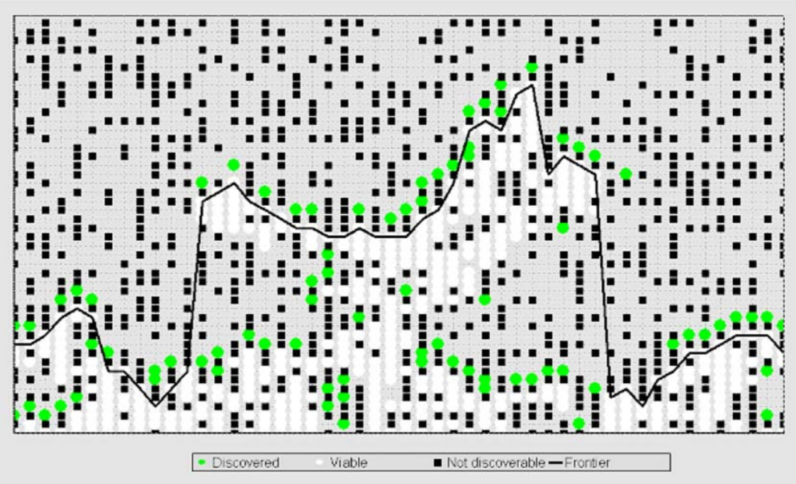

Fig. 9. Screen shot of computer-generated technology lattice showing mushroom-type development of the graph of viable sites in a typical run. Notice that even for the utilized search radius of 2, innovation jump sizes are unbounded at the cliff edges.

We begin by asking whether a more ‘crafts’ or a more ‘scientific’ search procedure will be more successful in maneuvering through the percolation maze. By crafts we mean a search for new techniques close in technology space to existing practice, i.e., small values of the search radius m. Scientific search by contrast looks farther a field and uses larger values of the search radius. To make this comparison fair, we hold the total R&D effort constant per BPF site and thus let the search probability scale downwards with increasing search radius. To this end we perform a grid search over a range of values of m and q, the percolation probability of the lattice, reflecting the density of seeding of the space with potential technologies. To control for the statistical variability of the runs, we report the results of 10 runs for each vector of parameter variables, differing only in the random seed used to initialize the random number generator.

In Fig. 10, we summarize the results of the grid search for the mean height of the frontier attained after 5000 time periods for a range of values of q and m and averaged over the 10 runs.[3] The range for q is centered around the critical percolation probability ($q = 0.593$) for the full-plane (and thus presumably near the critical value for our half-cylinder) and the search radius m is stepped from one to 20. We see that the average height attained increases (strongly) with q and (weakly) with m. However, the increase with search radius shows a decreasing slope, which also differs between

[3] Note that we do not consider here the unevenness of the BPF (which could be measured by its standard deviation), reflecting the variability of technical change across technology categories. The non-documented analysis of this indicator suggests that higher values of the search radius increase unevenness of the frontier, although at a decreasing rate.

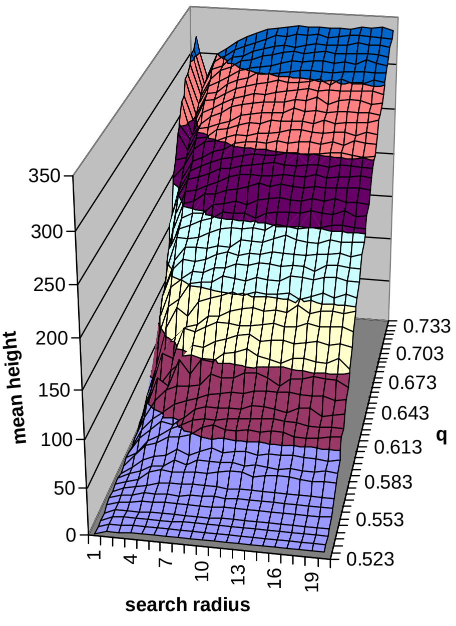

Fig. 10. The mean height of the BPF attained after 5000 periods as a function of the search radius and percolation probability q.

levels of q, and shows an, at first sight, surprising trough at $m=2$ for high values of q. Thus, gains exist for more farsighted R&D strategies, but they are not monotonic (for high values of q) and also saturate.

Fig. 11 shows the number of runs out of our sample of 10 for each parameter setting that *deadlock*, i.e., reach a state in which no further advance is possible even in principle, before the 5000 periods are over. At low values of q, the tendency for deadlocks to occur is high, irrespective of the search radius. This is obviously due to the fact that infinite or even large finite clusters will not exist for such low values of q. When the critical percolation probability of 0.593 is approached, the probability that at least a large finite cluster exists becomes higher, and deadlocks become less frequent. For values of q (slightly) above the threshold, given that the width of our lattice is finite, the large finite or even infinite cluster may not intersect the baseline, in which case it will be impossible to find with our search procedure. In summary, around the critical value, the probability of becoming deadlocked declines rapidly with the search radius m. For $m=1$, the number of deadlocks remains greater than zero for a value up to almost 0.7.

The occurrence of such a large number of deadlocks for values of q larger than the critical probability at first glance seems somewhat paradoxical. To see this, assume the search probability p to be equal to one. In this case, our search procedure will track the infinite cluster with certainty, irrespective of the search radius. However, with the search probability p smaller than one, this is no longer the case for the following

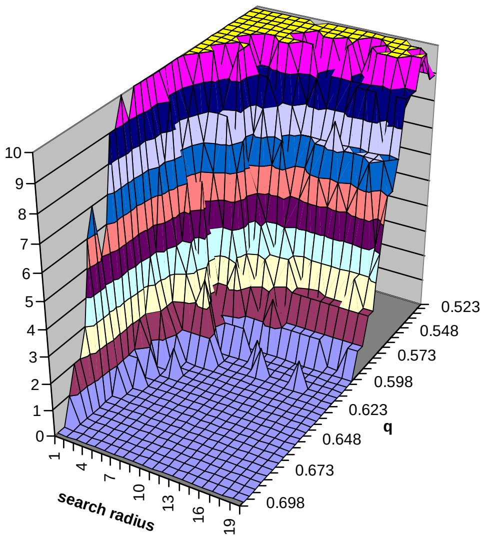

Fig. 11. Number of deadlocked runs out of 10 as a joint function of the search radius and the percolation probability q.

reason. Assume that a BPF site has reached a branching point on the infinite cluster, with one branch leading to a cul-de-sac while the other continues on the backbone of the cluster. If, for $m=1$, it chooses by chance the wrong branch, in the next time period the 'right' branch is already out of reach and can no longer be tested, regardless of how often we repeat the trial. For $p=1$ this problem disappears, since the system will always take both branches. For higher values of m the problem also declines in severity since even if the system takes the wrong branch, until it has pushed the BPF $m-1$ steps down the wrong branch, there is still a chance to discover the right branch before it is too late. Thus probabilistic search and shortsightedness introduce extreme path-dependence into the R&D process: the system can easily become trapped in a cul-de-sac even if a path of continuing technical progress exists.

This decline in the probability of deadlocking with increasing m would seem to be in contradiction to the trough in the mean height for high values of q and small m apparent in Fig. 10. The trough seems to be due to the countervailing influence of another factor, namely duplication of R&D effort. Larger search radii decrease the probability of becoming deadlocked, but for higher values of q this becomes less and less likely anyway, since the infinite cluster becomes an increasingly large proportion of all clusters (this follows from Fig. 5) and indeed of the entire lattice. The amount of duplication of R&D search, however, increases with the search radius m. This can easily be seen by considering two adjacent points on the BPF that are also lattice neighbors (of course they need not be lattice neighbors at all, but on average will be close). The area of the intersection of their m-neighborhoods will be an increasing share of the area they jointly cover (starting with $\frac{1}{8}$ at $m=1$, $\frac{4}{9}$ at $m=2$, and going to 1 as $m \to \infty$). This duplication of effort means that a given rate of investment in R&D

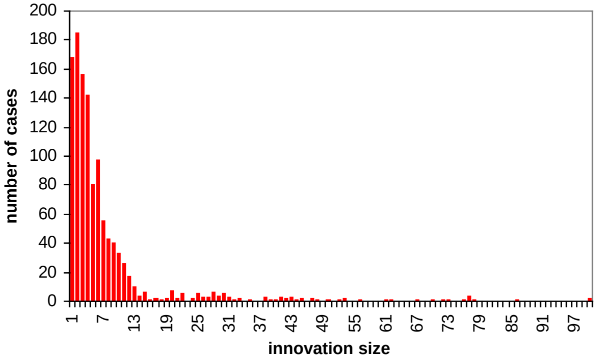

Fig. 12. Size distribution of innovations, $q = 0.603$, $m = 10$.

search will produce less technical change, i.e., advancement of the BPF, than otherwise. On balance, therefore, in a rich technological environment (q high compared to the critical percolation probability), increasing duplication at first outweighs the benefits of foresighted search and thus lowers the average rate of technical change. In a sparse technological environment (q near the critical probability), in contrast, where local dead-ends abound, the long-term benefits of foresighted search strongly outweigh any losses from duplication for all values of the search radius. In terms of our metaphor of crafts vs. science-based search, this suggests that although the science approach (large search radius) is ultimately superior in every technological environment (value of q), environments with relatively rich technological opportunities 'bifurcate' into crafts and scientific basins, separated by a barrier. By imposing some sort of meta-search dynamic on the radius of search (until now assumed exogenous), one could imagine cases of lock-in to one or the other of these basins (historically perhaps the crafts basin) or switching between them depending on agents' time-dependent perceptions about the technological environment in which they are operating.

These results should be compared with other algorithms for searching multidimensional spaces, such as genetic algorithms (cf. Goldberg, 1989), simulated annealing (cf. van Laarhoven and Aarts, 1987), and models of hill climbing on NK-landscapes (cf. Kauffman, 1993; Altenberg, 1997). Our notion of 'deadlocks' roughly corresponds to the well-defined concept of local optima in landscape models. The search radius is somewhat analogous to temperature in simulated annealing in the way it affects the probability of becoming trapped in a finite branch of the infinite cluster.

The size distribution of innovations can be examined by defining an innovation as any change in the height of a BPF site, and its size as the number of vertical sites covered by the change in one time period. In Fig. 12 we present a raw innovation size distribution, with the corresponding rank-order distribution of the same data in Fig. 13, in the same way as was done for the Trajtenberg patent citation data in Figs. 2 and 3 above. While it is clear that the distribution is highly skewed, the rank-order distribution does not quite conform to a power law. We generated these plots for a wide range of parameter values (q and m). Generically, the curves seem to be either

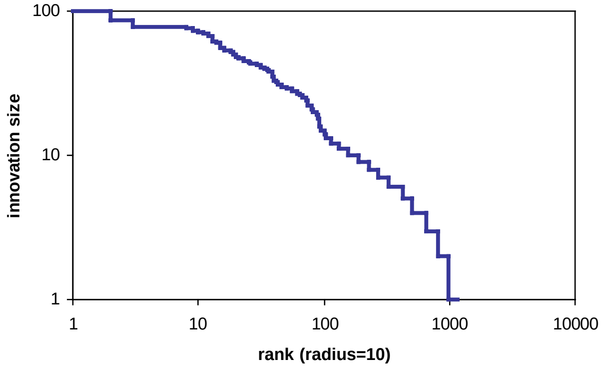

Fig. 13. Rank-order innovation size distribution, $q = 0.603$, $m = 10$.

linear, or concave in segments. This would imply that these distributions lie somewhere between a Pareto and lognormal distribution.

Are the large jumps or sudden gains in performance corresponding to the rightmost part of Fig. 12 consistent with the record of technological history? We think they are, even if they are relatively rare. Consider the speed of communication over long land distances or across oceans, for example. Until the advent of the telegraph, this speed was stagnant for millennia, limited by the speed of overland horse-drawn transport or sailing ships to less than 10 km/h.[4] Then within a very short period, albeit with somewhat limited bandwidth, it jumped to nearly the speed of light with the advent of the telegraph. This came about not through advances in materials transport but rather due to progress in a seemingly unrelated field, electricity. This kind of leapfrogging corresponds to the jumps induced in the BPF by overhanging cliffs in our percolation space, the height of which is in principle unbounded.

5. Refinements of the model and conclusions

The combination of a percolated technology search space with neighborhood-based probabilistic search enables us to endogenize the creation of technological trajectories and recover some of the characteristic (statistical) properties of the innovation process. In the simulation analysis so far, two major finding stand out. First, our model reproduces the skewed and possibly heavy-tailed distribution of innovation sizes known from the literature but for which a convincing theoretical explanation has been lacking until now. Skew distributions that generally look like the ones observed in the data on innovation returns or patent citations (as an indication of the technical or commercial value of a patent) emerge from our model in all parts of parameter space that we

[4] We neglect the historically rather limited role of visual semaphore systems over long distances such as were employed in France and England at the end of the 18th century. Their diffusion was undoubtedly limited by logistic factors and the necessity and unreliability of frequent relaying, and is impracticable over large bodies of water.

have investigated. A comparison of the precise statistical properties (such as whether or not heavy tails exist) of the simulated and empirical distributions is the subject of our current research.

A second finding of the model is that the efficiency of different search strategies varies systematically with technological opportunities. Innovation is revealed to be a highly path-dependent phenomenon in which excessive myopia can be a dangerous thing, since it can trap the system in dead ends. In environments with relatively low technological opportunities, increasing the search radius around the BPF (i.e., a 'science-based' mode of exploration aimed at covering large parts of technology space rather than in-depth search of small ranges) leads to a monotonic increase in the overall rate of progress, but this is not the case in environments with rich technological opportunities. There, the overall rate of technological progress first decreases with increasing search radius. Only at higher levels of the search radius does it recover and finally exceed the performance of the 'crafts-based' search strategy. The mechanism underlying this finding seems to be the interplay between duplication of research effort and the probability of getting locked into dead ends of the technology space.

There are various ways in which one could extend the model and the analysis of the simulation results. Until now we have assumed that R&D effort is uniform all along the BPF. This is not exactly a very rational hypothesis for agents who are aware of the historical rates of advance of different parts of the frontier. For example, a section of the frontier may be blocked by an impregnable barrier of impossible sites. It would gradually become obvious that continuing to invest in R&D along this section after many cycles of stymied advance would be pointless. Progress would only be possible by making an 'end run' around the barrier and taking a detour through a more fertile technological region, such as electricity proved for communications and computation. Thus a mechanism to shift R&D effort from stagnant areas to progressive ones would probably make sense, and might even be discoverable by agents with learning capabilities. One scenario for realizing this reallocation of R&D effort would be to scan for discontinuities in the BPF. If they exceed a threshold value, such as the search radius m, R&D effort is shifted from backward sites to the breakthrough sites.

The effect of learning in terms of the shortening of paths remains to be studied. We have also begun to study the time pattern of major innovation arrivals (major meaning larger than some threshold value), and preliminary results confirm one of the main aspects of the empirical record, namely overdispersion.

While such critical parameters as the level and distribution of R&D effort along the BPF (which are assumed to be uniform and constant in this version) and the search radius could be endogenized in an agent-based feedback extension of the model, the percolation probability q presents a more fundamental difficulty. We have taken it to be a constant of nature until now, but what is its appropriate value? Below the critical value technical change will eventually come to an end. Just above it, the infinite cluster possesses some interesting and potentially intriguing properties: the cluster is geometrically a fractal and the distribution of finite-cluster sizes obeys a power law. This clearly has implications for the innovation size distribution, as we have seen. As q approaches one, however, the world becomes in some sense increasingly 'regular'. For example, the innovation size distribution seems to increasingly resemble a lognormal.

The empirical evidence is ambiguous on this point until now (see Silverberg and Verspagen, 2003b). Thus one approach would be to tune or calibrate the model, using q, against empirical data. How this can be done is not quite clear (and would also require a better empirical analysis). Another possibility would be to eliminate the exogenous and uniform nature of q, and instead adopt the NIP assumptions and allow the system to self-tune in the sense of self-organized criticality (SOC).[5] This would mean that R&D effort would increase and decrease through a feedback loop to the level that would just allow progress along the BPF to overcome the weakest obstacle, however high it is, as in the models of social percolation or the Bak–Sneppen model of co-evolution. The properties of such a model remain to be studied, but our intuition suggests that the effective value of q would hover most of the time just above the critical one without having to impose this exogenously.

In conclusion, we can only restate our delight to find that the formulation of a simple 'complex dynamics' models based on very few assumptions derived from the stylized facts of innovation discussed in the literature already generates a range of key phenomena known to characterize innovations but until now regarded as unexplained and separate, such as technological trajectories, highly skewed and possibly Pareto-distributed innovation size distributions, temporal and spatial clustering (discussed in Silverberg and Verspagen, 2003b), and interesting search-theoretic properties. Percolation of a multidimensional lattice or complex directed graph, in association with local stochastic search and the requirement that viable technologies be linked backwards in time and space, seems to provide an ideal framework for naturally integrating these disparate observations. In addition, this work opens up a fruitful trajectory of extensions in the direction of agent-based R&D models and self-organized criticality.

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[5] Cf. Bak (1996). On the relationship between SOC and percolation, see Grassberger and Zhang (1996) and Sornette and Dornic (1996). Extremum dynamics along the lines of the Sneppen (1992) model might be an appropriate avenue of attack.

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